Double Integrals

Problem 1 (Stewart, Exercise 15.1.11). *Evaluate the double integral*

$$\iint_R (4-2y)\,dA, \quad R = [0,2]\times[0,1],$$

by first identifying it as the volume of a solid.

Problem 2 (Stewart, Exercise 15.1.34). *Evaluate the double integral*

$$\iint_R \frac{1}{1+x+y}\,dA, \quad R = [1,3]\times[1,2].$$

Problem 3 (Stewart, Exercise 15.1.42). *Find the volume of the solid in the first octant bounded by the cylinder $z = 16 - x^2$ and the plane $y = 5$.*

Problem 4 (Stewart, Exercise 15.1.49). *Use symmetry to evaluate the double integral*

$$\iint_R \frac{xy}{1+x^4}\,dA, \quad R = [-1,1]\times[0,1].$$

Problem 5 (Stewart, Exercise 15.2.22). *Evaluate the double integral*

$$\iint_D y\,dA,$$

where D is the triangular region with vertices $(0,0)$, $(1,1)$, and $(4,0)$.

Problem 6 (Stewart, Exercise 15.2.27). *Find the volume of the tetrahedron enclosed by the coordinate planes and the plane $2x + y + z = 4$.*

Problem 7 (Stewart, Example 15.2.4). *Find the volume of the tetrahedron bounded by the planes $x + 2y + z = 2$, $x = 2y$, and $z = 0$ using:*

(1) double integrals,
(2) triple integrals.

[Hint: The tetrahedron lies in the first octant. Sketch it by looking from your left.]

Problem 8 (Stewart, Exercise 15.2.40). *Sketch the solid whose volume is given by the integral*

$$\int_0^1 \int_0^{1-x^2} (1-x)\,dydx.$$

Problem 9 (Stewart, Exercise 15.2.(51,52,55)). *Evaluate the following integrals:*

(1) $\int_0^1 \int_{3y}^3 e^{x^2}\, dxdy$,

(2) $\int_0^1 \int_{x^2}^1 \sqrt{y} \sin y \, dydx$,

(3) $\int_0^1 \int_{\arcsin y}^{\pi/2} \cos x \sqrt{1+\cos^2 x}\, dxdy$.

[Hint: Reverse the order of integration first.]

Problem 10 (Cal Final, Summer 2018W). *Calculate the iterated integral*

$$\int_0^4 \int_{\sqrt{x}}^2 e^{y^3}\, dydx.$$

Problem 11 (Stewart, Exercise 15.2.(65,67,68)). *Use geometry to evaluate the following double integrals.*

(1) $\iint_D (x+2)\, dA, \quad D = \{(x,y) \mid 0 \le y \le \sqrt{9-x^2}\}$,

(2) $\iint_D (2x+3y)\, dA, \quad D = [0,a] \times [0,b]$,

(3) $\iint_D (2 + x^2y^3 - y^2 \sin x)\, dA, \quad D = \{(x,y) \mid |x| + |y| \le 1\}$.

Problem 12 (Stewart, Exercise 15.4.9). *Find the mass and center of mass of the lamina that occupies the region bounded by the curves $y = e^{-x}$, $y = 0$, $x = 0$, and $x = 1$ if its density function is $\rho(x,y) = xy$.*

Problem 13 (Stewart, Exercise 15.5.3). *Use the formula*

$$A(S) = \iint_D \sqrt{1 + f_x^2 + f_y^2}\, dA$$

to find the surface area of the part of the plane $3x + 2y + z = 6$ that lies in the first octant.

Problem 14 (Stewart, Exercise 15.5.7). *Find the surface area of the part of the hyperbolic paraboloid $z = y^2 - x^2$ that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$. [Hint: Use the formula for surface area via double integrals.]*

References

[1] J. Stewart: *Calculus* 8th Edition, Cengage Learning, Boston 2016.